From Tactics to Structure Editors for Proofs

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What are tactics?

Two main types of UIs for theorem proving:

- direct style: proofs are programs, so we write them just like we write programs (Agda, Idris).
- tactics: closer to how we reason about things on paper and how we think about reasoning, e.g. recursive proof term is "induction" not "recursion" (Coq, Lean, Idris *Pruviloj*).

What is my talk about?

- User interfaces for theorem provers & program editors
- How can we use *tactics* to improve theorem prover & editor UIs?

Motivation of this talk: Coq nightmare

```
Definition balanceL {nl ml d cl cr nr mr} (p : color) (l : near_tree nl ml d cl) (r : tree nr mr d cr) :
  color_ok p (fix_color l) (* important claim! *) ->
  color_ok p cr ->
  {tr : near_tree (nl + nr) (ml + mr) (inc_black d p) p | dflattenn tr = dflattenn l ++ dflatten r}.
Proof.
  destruct l as [s1 o1 s2 o2 s3 o3 d' x y z | s o d' c' cc l'].
  (* l is bad *)
  + case: p => //= cpl cpr.
    rewrite -(addnA (s1 + s2)) -(addnA (o1 + o2)).
    exists (Good Black (rnode (bnode x y) (bnode z r))).
    by rewrite /= !catA.
  (* l is good *)
  + case: p => /= cpl cpr; last by exists (Good Black (bnode l' r)).
    case Hc': c' in cpl.
    (* bad pattern (c' and p are red) *)
    - destruct l' as [|s1 o1 s2 o2 d cl' cr' c' w1 w2 l'1 l'2] => //.
      subst c'; destruct cl', cr', cr => //.
      exists (Bad l'1 l'2 r).
      by rewrite /= !catA.
    (* otherwise *)
    - subst c'; destruct cr => //.
      by exists (Good Red (rnode l' r)).
Defined.
```

Sometimes, dependently-typed programming can be quite tricky and dirty:

- byzantine invariants;
- case bloating.

Problems with each approach

- Direct style: little automation available (boring pattern-matching, boring code).
- Tactics: little idea of what you're actually doing; bad for actual programming.

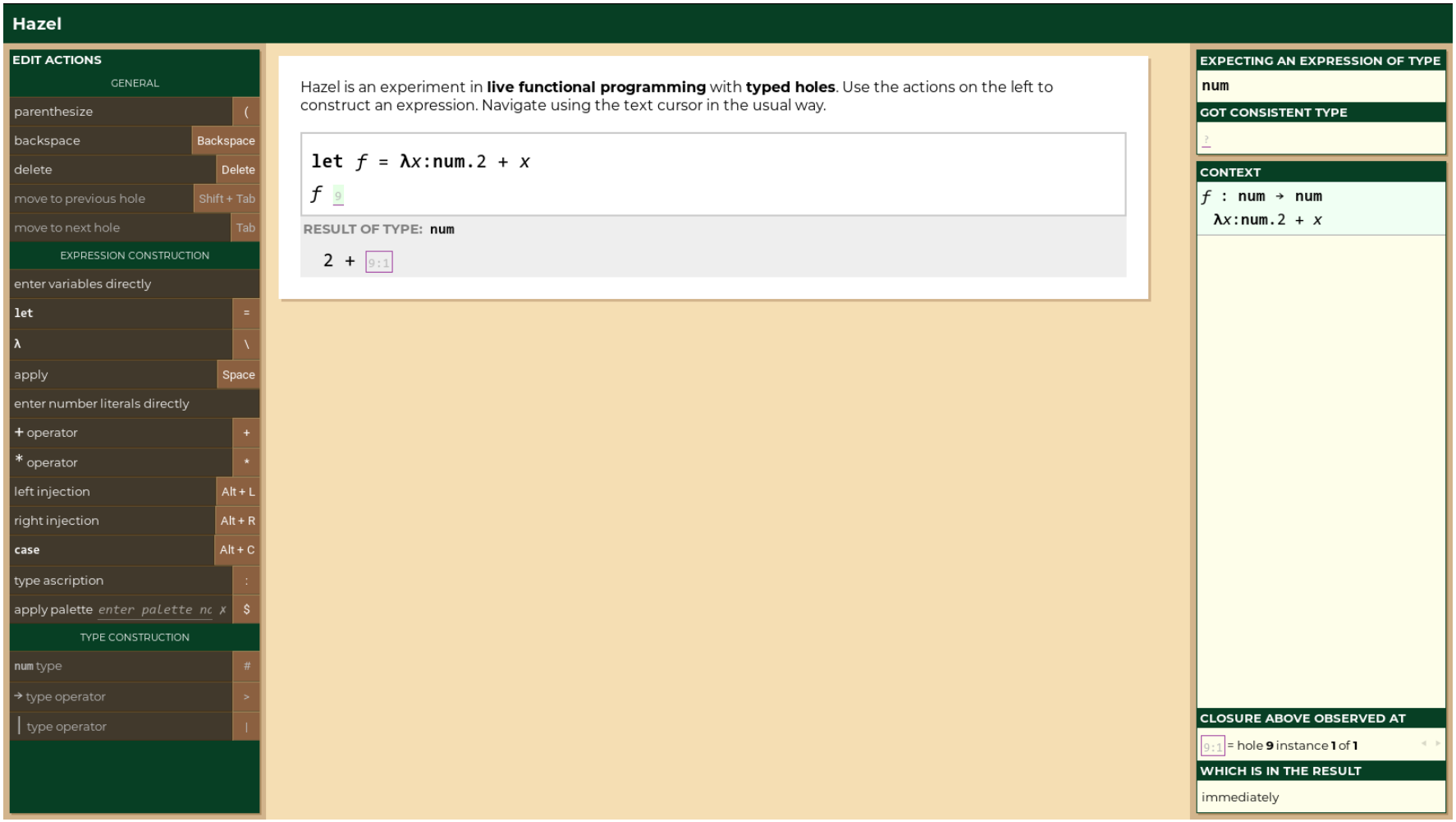
My proposal

- Design new program editors that integrate tactics as an automation method
- Design new theorem provers that allow mixing direct-style & tactics
- Study formal semantic foundations for tactics

Proposal 1: use tactics as automation

- Help automate case-splitting, especially in presence of indexed datatypes/GADTs (`induction`)
- Writing trivial clauses automatically using program search/synthesis (à la `intuition` & `crush`)
- More complex tactics might also find their uses in programming, with or without dependent types.

Holes can also help here



Proposal 2: combining tactics & programs

- Happens very often when programming with complex types: not all details we care about
- Automate chores using edit-time tactics and hide them in local, collapsable holes
- Challenge: actually generate good code
- But, do not *erase* tactics from the program: even if other parts of the program change, the housekeeping can stay the same

We may want to write programs like this...

```
Fixpoint filter {n A} (p : A -> bool) (v : vec A n) : vec A (count p v) :=
  match v with
  | VNil _ => VNil
  | VCons h t => if p h
                 then VCons h (filter p t)
                 else filter p t
  end.
```

[does not actually work in Coq]

```
filter : ∀ {n A} → (A → Bool) → Vec n A → ∃ (λ n' → Vec n' A)
filter p [] = (0 , [])
filter p (h :: t) with p h
... | true with filter p t
...                | (n'' , t') = (suc n'' , (h :: t'))
filter p (h :: t) | false = filter p t
```

But what we really want is something more like this...

```
Fixpoint filter {n A} (p : A -> bool) (v : vec A n) : vec A (count p v) :=
  match v with
  | VNil _ => VNil [ proof ]
  | VCons h t => if p h
                 then VCons h (filter p t) [ proof ]
                 else filter p t [ proof  ]
  end.
```

Proposal 3: understand the semantic foundations of tactics

- Tactics are transformations on open terms
- We should study the semantics of those transformations formally
- Benefits: create tactics that compose well, prevent tactics from breaking valid proofs/programs (like Ltac)

Building on previous work

- Hazelnut paper (POPL '17) gave semantics of some very basic "tactics" (term insertion, moving to other holes)
- Open problem: design a language to program well-behaved tactics

Putting it all together

- Traditional sorce code based editors are inadequate in a “typed” world; too much focus on minor details requireds
- Tactics can help programmers automate away those chores and focus on writing good and correct programs!
- I propose using tactic-enhanced program editors, with semantically sound tactics, to write programs.
- More automation, more editor assistance, less bookkeeping, better programs.