Type-Driven Development of Certified Tree Algorithms

An Experience Report on Dependently-Typed Programming in Coq

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Our story

Dependently-typed Programming in Coq: Why and How?

The "Why" Why did we use dependent types in Coq? Under what occasions are they useful?

The "How" What is the best approach towards dependently-typed programming in Coq? How did we approach it?

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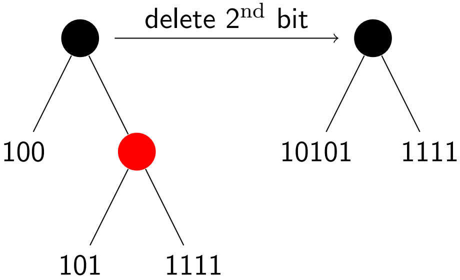
Quick recap: what were we working on?

Bit vectors with efficient insert & delete:



- Represented using a red-black tree
- Insertion and deletion might involve inserting/deleting nodes

The motivation

Deletion from red-black trees is too hard.

- long standing problem with a few proposed solutions [Kahrs 2001; Germane & Might 2014], but none of them totally satisfactory for us;
- complex invariant hard to describe precisely
- difficult to transcribe to our non-standard tree structure (bit-borrowing, leaf merging, etc.)

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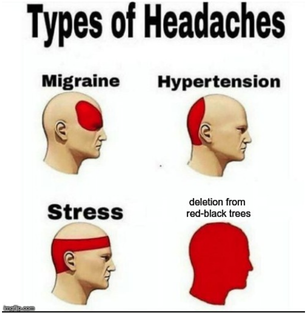

Using dependent types

Problem:

- not sure about how to do case analysis;
- not sure about the exact invariants;
- not sure about the auxiliary structures required.

Idea: use dependently-typed programming to guide programming process.

Auxiliary structures?

The intermediate data structures required for re-balancing the tree:

```
Inductive near_tree : nat -> nat -> nat ->
  color -> Type :=
| Bad : forall {s1 o1 s2 o2 s3 o3 d},
  tree s1 o1 d Black ->
  tree s2 o2 d Black ->
  tree s3 o3 d Black ->
  near_tree (s1 + s2 + s3) (o1 + o2 + o3) d Red
| Good: forall {s o d c} p,
  tree s o d c ->
  near_tree s o d p.
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Re-balancing requires temporarily breaking the red-black tree invariants, hence the need for auxiliary structures.

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Take 2: Ltac

- use *tactics* to develop the program
- we ascribe strict types to each function, allowing to be completely sure that our code is correct
- as a side effect, we got a very clean specification
    - no “external” lemmas which can be easy to forget
    - all desired invariants were encoded into the types

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Programming Coq with tactics

The Pros

- “No brainer”: no need to fully understand the algorithm [Chlipala 2013]
- Easy to refactor: when underlying data structures change
- Quick fixes & adapting to changes

Programming Coq with tactics

The Cons

- You don't know what you're actually doing
- Readability: other people don't know what you're doing
- Semantics of Ltac changes frequently

Type-driven development?

"Type-driven" in what sense?

Regular development: design the algorithm, and then write types to check that you're correct.

Type-driven development: write types to declare what you want, and then code until it type checks

It type checks, ship it!

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Applying the TDD methodology

At first, we had no clue about what the delete algorithm should look like!

We began with a complete specification:

```
Definition ddelete
  (d: nat)
  (c: color) (num ones : nat)
  (i : nat)
  (B : tree w num ones (incr_black d c) c) :
  { B' : tree (num - (i < num))
    (ones - (daccess B i)) d c |
    dflatten B' = delete (dflatten B) i }.
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Applying the TDD methodology

Finding the missing auxiliary structure

We started to develop our function and found out that we needed to keep track of whether the height of a node has been decreased:

```
Inductive del_tree : nat -> nat -> nat -> color ->
  Type :=
| Stay : forall {num ones d c} pc,
  color_ok c (inv pc) -> tree w num ones d c ->
  del_tree num ones d pc
| Down : forall {num ones d},
  tree w num ones d Black ->
  del_tree num ones d.+1 Black.
```

Applying the TDD methodology

Refining the type

Now, we can write specifications for helper functions as well:

```
Definition balleft {lnum rnum lones rones d cl cr}
  (c : color)
  (l : del_tree lnum lones d cl)
  (r : tree w rnum rones d cr)
  (ok_l : color_ok c cl)
  (ok_r : color_ok c cr) :
{ B' : del_tree (lnum + rnum) (lones + rones)
         (incr_black d c) c |
         dflattend B' = dflattend l ++ dflatten r }.
```

Iterative development process similar to using holes and case-split iteratively in Agda or Idris.

Extraction

Two types of extraction:

- Extracting ML code from code defined using tactics
- “Extracting” a non-dependently-typed core of the algorithm within Coq (see ITP talk tomorrow)

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Take 3: rewrite using `Program`

`Program` is a framework for dependently-typed programming in Coq [Sozeau 2006; 2008].

- Cleaner code: automatically generate type coercions for terms

Experiences with `Program`

The Bad

- Many problems with unification engine: existential variables caused a lot of problems
- `program_simpl` was too aggressive sometimes, destroying goals in the process
  - Solution: disable `program_simpl`, unless the goal was directly solved by it.
- Bad error messages and mysterious failures
  - `Error: the kernel does not support existential variables`
  - Workaround: explicitly match on each argument that needs to be matched
- Performance issues with `Program`
- Simplifying and rewriting

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- Readability and writability
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- Non-structural recursion using `measure`

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The other alternative: `Equations`

Dependent pattern-matching compiler for Coq [Sozeau 2010; Sozeau & Mangin 2019].

- Even more readable code (Agda-like)
- `funelim` tactic supports easy pattern-matching in proofs
- No more Axiom K [Sozeau & Mangin 2019]

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e.g. issues #195, #212, #216, #217, #81 (closed), #179 (closed)

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Our takeaways

- Dependent types in Coq are often useful. Use them as you see fit!
- Type-driven development is a great way to write programs that you don't know how to write!
- Tactics can be used to write programs, quite reliably
- Coq community needs to look at dependent types more (fix bugs, develop tools, etc.)

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- Erasure of type indices à la [Brady, McBride & McKinna 2003]
- Showing only computationally-relevant terms
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Final remarks

Come to our ITP talk tomorrow at 16:30!

R. Affeldt, J. Garrigue, X. Qi, K. Tanaka. “Proving Tree Algorithms for Succinct Data Structures”.

https://github.com/affeldt-aist/succinct